SREB

Early Math Matters

Factoring in Teacher Knowledge and Practice

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Southern Regional Education Board

SREB.org

Math achievement is critically important to the changing global economy. In order to be successful, students must develop a strong foundation in mathematics early in their school careers. But the training elementary teachers receive doesn't always prepare them to teach math well, and math anxiety can keep both students and teachers from reaching their full potential.

This report explains why early math learning is so important, the current state of math instruction, issues with elementary teacher preparation and professional development, and how math anxiety impacts achievement. It also presents recommendations state leaders can use to help raise the math achievement of their students.

Inside

Why Math Matters

State policies and public perception recognize how important it is that students read proficiently by the end of third grade. This recognition may imply to some that reading should be the primary focus of the elementary years. But the truth is that no one subject is paramount, and math should not fall by the wayside during children's first years in school. While reading is critical to future academic and personal success, so are a strong foundation in numeracy and the analytic skills children learn by tackling mathematical problems. In fact, some studies show that early math skills are better predictors of students' later academic success than are early reading skills.

Individuals hold different ideas about the ultimate goal of mathematics instruction, especially early in school. Mathematics professor Dr. Robert H. Lewis considers math the most misunderstood subject, as he describes in this adapted excerpt from his essay:

> *Mathematics is not about answers.* It is about building the ability to think, perceive and analyze. One man once said to me, "You know, I had to memorize the quadratic formula in school, and I've never once done anything with it. I've since forgotten it. What a waste!" If I had been the man's first grade teacher, would he have said, "You know, I can't remember anymore what the name of Dick and Jane's dog was. Therefore, you wasted my time when I was six years old?" Of course not! we understand intuitively that the details of the story were not the point. It was to learn to read, which opened new vistas of understanding and many other competencies. The same is true of mathematics.
>
> Education is a deep, complex and organic representation of reality in the student's mind — made of concepts, not equations or memorized information. Teaching mathematics, like teaching any other subject, is not a matter of pouring knowledge from one mind into another. **It is one candle igniting another.**

Unfortunately, Lewis' view of math is not necessarily reflected in the instruction students receive. Dale Parnell wrote just over 20 years ago that "the study of mathematics stands, in many ways, as a gateway to student success in education." He was referring to math instruction that tended to isolate knowledge and application, leading to students who saw "little personal meaning" in what they studied in their K-12 math classes. Parnell called for math instruction to stress application and connect more directly to real-life situations and the skills students need for a changing workforce.

The reality is that math achievement is critically important to the changing global economy. In 2012, the National Research Council published *Education for Life and Work*, a consensus report explaining how schools could better help students develop the skills needed for success as citizens and in the workforce. The report explains that many competencies central to the study of mathematics, including reasoning, critical thinking and problem solving, are also important 21st-century skills. These skills are vital for adults, both in daily life and in their careers — whether they be carpenters or doctors.

This report was prepared by Samantha Durrance, policy analyst, under the leadership of Jeff Gagné, director of policy analysis, and Joan Lord, vice president of education data, policy research and programs.

SREB's 2019 report *Unprepared and Unaware* pointed out that workforce changes have already resulted in a gap between the number of middle-skill jobs available and the number of workers who are qualified to fill them. These jobs often pay well but require education and training beyond high school — education and training that equips workers to work closely with machines and adapt to technological advancements. Students who have trouble earning high school-level math credits in courses like algebra likely struggle with the 21st-century skills they need to be competitive in the workforce, too. But the roots of these struggles begin well before high school.

When Mathematical Understanding Begins

The development of math skills begins long before children enter school. Researchers including Starr, Libertus and Brannon have found that infants as young as six months old demonstrate a basic sense of number that enables them to compare two quantities and decide which is larger. Called the "approximate number system," this earliest form of numeracy is believed to be the foundation for the more exact sense of number that children develop through formal education.

Experiences in early childhood with the concept of number — like counting objects with a parent — affect the skills children later bring to school. Jordan, Kaplan, Ramineni and Locuniak wrote in 2009 of a growing consensus among researchers that "many mathematics difficulties in elementary school can be traced to weaknesses in basic whole number competencies, that is, in understanding the meaning of numbers and number relationships." Signs of these weaknesses include inaccurate counting and computation and slow recall of mathematical facts. Children who engage in number activities and games with their families are likely to have better number competence and be better prepared for early success with math than children with fewer number-related experiences before they enter school. And these early experiences matter in the long run. According to a 2007 study by Duncan and colleagues, early math skills are actually a better predictor of overall academic achievement later on than are early reading skills.

Unfortunately, the math experiences children have before they enter school are often not adequately nurtured in the primary grades. As a result, students may not develop strong foundational math skills, making later learning more difficult. In 2009, the National Research Council's Committee on Early Childhood Mathematics examined research on how young children learn — and should learn — math. The resulting report, *Mathematics Learning in Early Childhood*, found that many early childhood settings did not provide adequate learning experiences in math, in part because early childhood teachers were not well prepared to provide math instruction. The committee concluded that for most children, "the potential to learn mathematics in the early years of school is not currently realized."

> "For most children, the potential to learn mathematics in the early years of school is not realized."

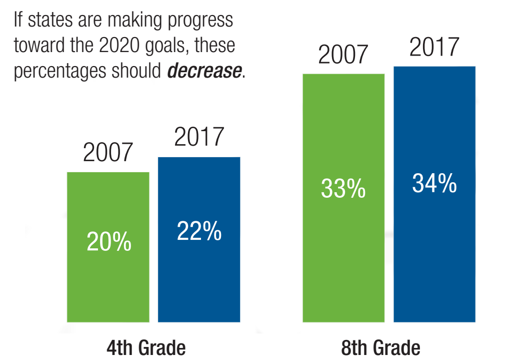
Researchers have known for decades that math learning in the United States does not fully capitalize on the capabilities of its children. A 2001 report by the National Research Council's Mathematics Learning Study Committee concluded that children too often receive inadequate math instruction in early childhood and throughout the elementary and middle grades. Ultimately, students end up ill-prepared for algebra and more advanced math topics in high school, which in turn makes math achievement more difficult in postsecondary settings. While the SREB region and the nation as a whole have made progress toward improving math education, they still have a long way to go to fully harness the capabilities of young children and prepare them for success in elementary school and beyond.

Math Achievement in SREB States

The status of math achievement in fourth and eighth grades in SREB states is very telling. In SREB's *Challenge to Lead 2020* goals for student achievement — a 2016 update to the goals originally adopted in 2002 — SREB called for 90 percent of fourth and eighth graders to score at or above the Basic level on the National Assessment of Educational Progress in math by 2020. It also called for the percentages of fourth and eighth graders who score at or above the Proficient level on NAEP in math to increase regularly, and ultimately exceed the national average.

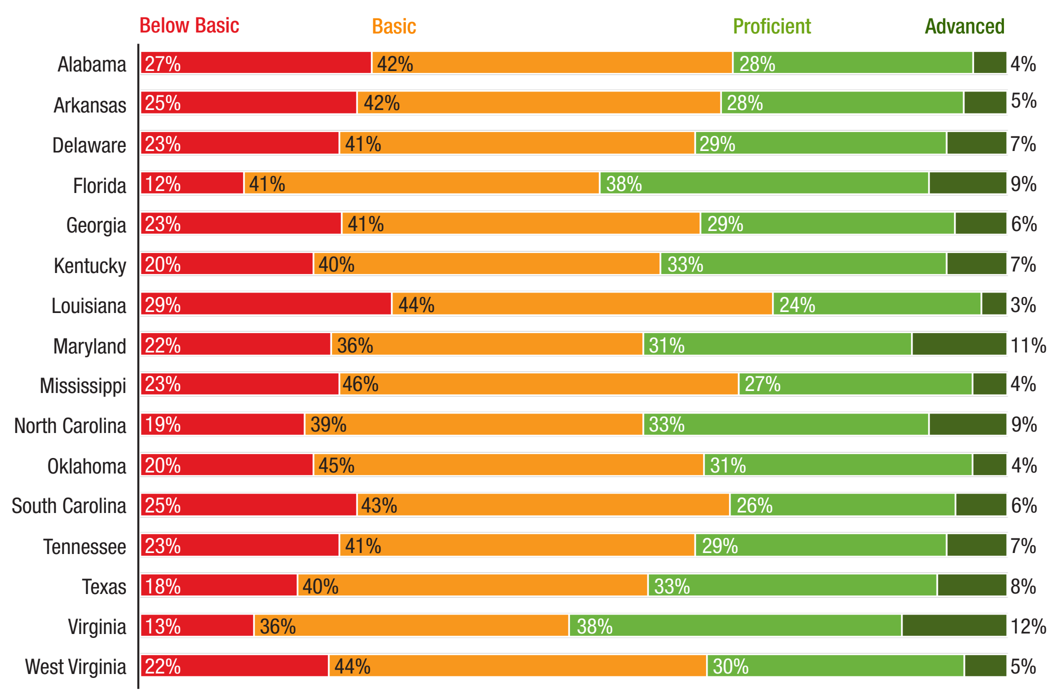
Known as the Nation's Report Card, NAEP is a key measure of academic achievement across states. Fourth grade NAEP math scores indicate the degree to which states are developing students' math skills in the early grades. Eighth grade math scores demonstrate how strong students' mathematical foundations are, and whether the middle grades build on those foundations in a way that prepares students for success in algebra by ninth grade.

Students Performing Below Basic in Math on NAEP



2017 Fourth Grade NAEP Math Results by Level, SREB States

All Students



Source: SREB, based on data from the National Center for Education Statistics

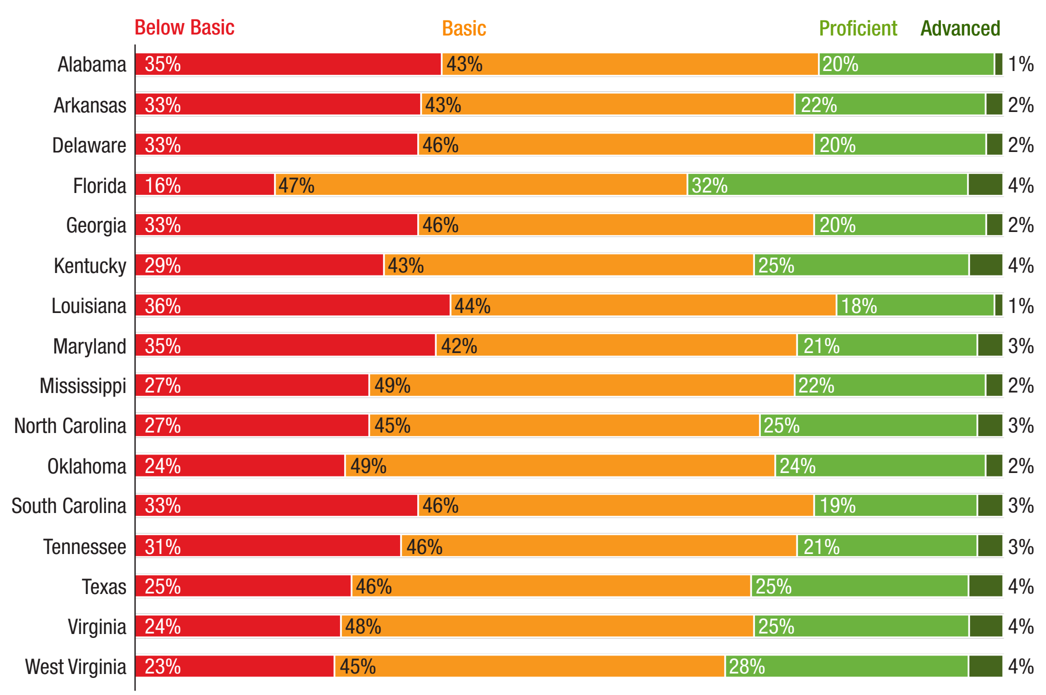
SREB states have made very modest progress toward improving math achievement before high school, as demonstrated by a 2-point increase from 2007 and 2017 in the percentage of fourth graders scoring at or above the Proficient level in the region. Eighth graders made a similar increase. Yet, while 36 percent of fourth graders in SREB states performed at or above Proficient in 2017, only 28 percent of eighth graders did. At the other end of the performance scale, the percentage of students performing below the Basic level on NAEP in math was 22 percent for fourth graders in 2017, but 34 percent — 12 points higher — for eighth graders. And each of these proportions grew slightly from 2007 to 2017, indicating that the SREB region was actually further away from some of the Challenge to Lead 2020 targets in 2017 than it had been 10 years earlier.

A closer look at the data reveals that the overall NAEP results hide a lot of important variation between student groups. In 2017, 36 percent of all fourth graders in SREB states performed at or above Proficient on NAEP in math. A much smaller proportion of fourth graders from low-income families met the Proficient benchmark: just 24 percent. And for English language learners and fourth graders with disabilities, the rates were even lower: 11 percent and 14 percent, respectively.

The story is even more alarming for eighth-grade students: while 28 percent of all eighth graders in the SREB region met the NAEP Proficient benchmark in math in 2017, this rate included just 16 percent of low-income students, 6 percent of English language learners, and 6 percent of students with disabilities.

2017 Fourth Grade NAEP Math Results by Level, SREB States

Students from Low-Income Families



Source: SREB, based on data from the National Center for Education Statistics

NAEP Achievement Level Definitions

Basic: Partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at the grade level assessed.

Proficient: Solid academic performance for the grade level assessed. Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.

Advanced: Superior performance for the grade level assessed.

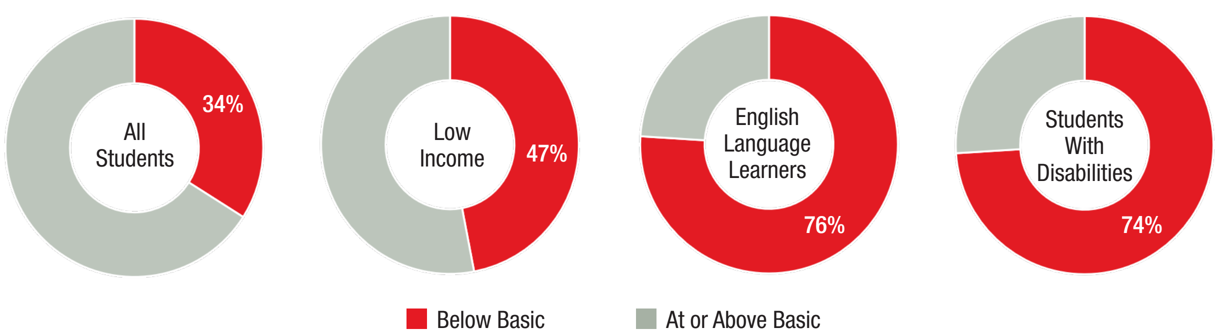
Source: National Center for Education Statistics. The NAEP Glossary of Terms. www.nces.ed.gov

What causes more eighth graders than fourth graders to perform poorly in math, and why do so few students master math before ninth grade? Mathematics concepts build on each other, and the foundation for middle grades and high school achievement in math is built — either well or poorly — in elementary school. If students don't develop a strong foundation during these years, later concepts become increasingly difficult to master. Research points to actions that can help reverse these trends.

First, teachers of math in the elementary grades need better mathematical content knowledge for teaching — not just the ability to *do* math, but also the knowledge needed to teach math in a way that helps students build a deep understanding of mathematical concepts. This content knowledge should be developed both in educator preparation programs and through ongoing professional development once teachers enter the classroom.

Second, teachers, parents and children must be encouraged to adopt a growth mindset toward math: a mindset that acknowledges and appreciates the importance of practice and effort for success. Building mathematical knowledge requires time and involves making mistakes. Understanding this can help individuals set reasonable expectations for themselves, remain confident in their ability to learn math, and experience less anxiety when they struggle.

Eighth Grade Students Performing Below Basic on NAEP in Math, 2017



Source: SREB, based on data from the National Center for Education Statistics

Where We've Grown:
Getting Learning Standards and Instructional Focus Right

Mathematics instruction in the United States has long been criticized as "a mile wide and an inch deep," a reference to the many math topics covered at each grade level and the shallow exploration of each. This observation of "breadth over depth" in math content first surfaced in a 1996 National Center for Education Statistics report on the Third International Mathematics and Science Study. The report noted that coverage of mathematical topics in U.S. eighth grade classes was less focused than in Germany and Japan, two international leaders at the time. This was in part because curriculum experts in the United States tended to recommend that math curricula cover more topics at the first- through eighth-grade levels than the international average. A study of classroom practices confirmed that eighth grade math classrooms in the U.S. covered an average of 1.9 topics per lesson, more than Germany or Japan, and with a greater variety of topics during a school year. When students are asked to learn more topics in the same amount of instructional time, something has to give — and that something tends to be mastery.

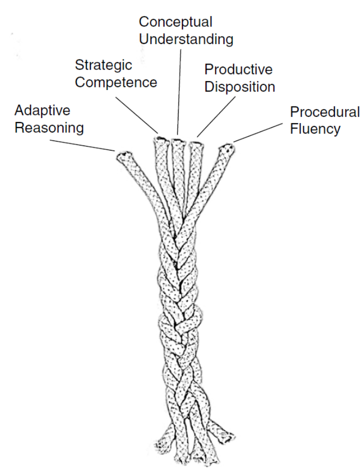
In 2001, the National Research Council's Mathematics Learning Study Committee published *Adding It Up*, a blue-ribbon report on pre-K to eighth grade math teaching and learning. The report found that students in the United States did well enough with rote computation, but they performed poorly on international assessments that require more advanced mathematical problem solving. Like the 1996 NCES study, *Adding It Up* noted that U.S. math textbooks often covered more topics, but more superficially, than textbooks in nations elsewhere in the world that performed well on math assessments.

Adding It Up also described the changes observed in mathematics teaching and learning in the United States during the latter half of the 20th century. The period saw several reversals in the focus of math instruction. Changes in society and the workforce in the 1950s and '60s led to an emphasis on understanding mathematical concepts, rather than just building computational skill. Another swing of the pendulum soon followed, and computational fluency ruled the day in the 1970s. In the '80s and '90s, a new movement again emphasized mathematical reasoning and problem solving, though not without critics who touted the value of computational fluency and fact memorization and would have preferred to target those skills.

The experts involved in the Mathematics Learning Study Committee that produced *Adding It Up* recognized that regularly changing instructional focus from one extreme to another was detrimental to math teaching and learning. Instead, they presented a comprehensive model of mathematical proficiency composed of five intertwined strands:

- **Conceptual understanding**, or comprehension of mathematical concepts, operations, and relationships;
- **Procedural fluency**, or skill in carrying out mathematical procedures flexibly, accurately, efficiently and appropriately;
- **Strategic competence**, or the ability to formulate, represent and solve mathematical problems;
- **Adaptive reasoning**, or capacity for logical thought, reflection, explanation and justification; and
- **Productive disposition**, or the inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

Intertwined Strands of Proficiency



Reprinted with permission from Adding it Up, 2001 by the National Academy of Sciences. Courtesy of the National Academies Press, Washington, D.C.

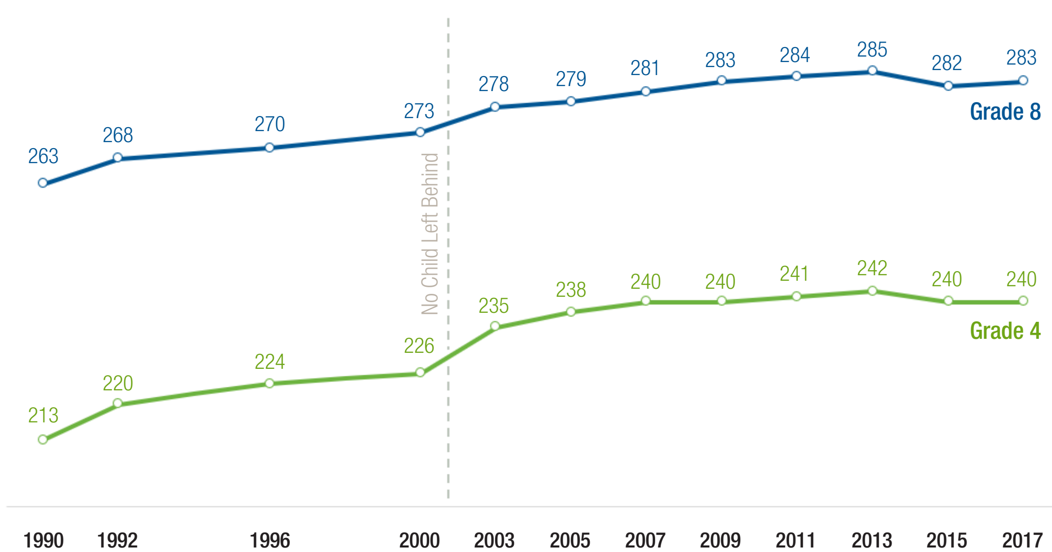
This model visualizes mathematical proficiency as multidimensional, requiring a variety of skills and understandings. Strong curricula and instruction must emphasize all the strands, as they all support each other. By including both conceptual understanding and procedural fluency, the committee also emphasized that computational fluency *with understanding* is necessary for mathematical proficiency. Neither is sufficient on its own. This recognition intended to put an end to the back-and-forth of instructional focus seen in previous decades.

> "Computational fluency with understanding is necessary for mathematical proficiency. Neither is sufficient on its own."

Policymakers took note of the findings of *Adding It Up*. Accountability- and standards-based education reforms in the first two decades of the 21st century have sought to hold schools formally responsible for student learning and increase the rigor of state standards across the nation. These initiatives have had mixed results. A 2011 analysis by economists Dee and Jacob concluded that the accountability systems created in response to 2001's *No Child Left Behind* law improved math proficiency in both fourth and eighth grade, with the greatest gains occurring in fourth grade. This can be observed in scale score gains on NAEP from the year 2001 onward.

Following No Child Left Behind, governors and other state leaders across the nation strove to create rigorous math and language arts standards for learning that would prepare all students for college and careers. They worked to make the new math standards more focused, tackling the criticism that mathematics instruction in the United States covers too many topics per grade level with too little depth. They also carefully considered and aligned the progression of standards from kindergarten to high school to ensure that concepts build on each other across grades.

Change in Average NAEP Mathematics Scale Scores for 4th and 8th Grade Students Over Time



Source: SREB, based on data from the National Center for Education Statistics

In recognition of the strategic competence, adaptive reasoning and productive disposition strands of proficiency proposed by *Adding It Up*, states created standards for mathematical practice to complement those for mathematical knowledge. The standards for mathematical practice outlined proficiencies and attitudes that students need to develop to be successful with math — for example, persevering in solving problems, constructing mathematical arguments and modeling everyday problems with mathematics. In many states, the new math standards also placed more focus on building a conceptual understanding of math than previous standards and were more rigorous, aimed at ensuring that all students graduate from high school ready for college or careers.

Overall, state math standards in the United States are stronger today than they were, and state standards for math proficiency are also higher, according to Hamlin and Peterson's 2018 analysis for EducationNext. Even so, better standards alone are not enough to improve math achievement. Standards-based education reform has so far had little apparent effect on student achievement in math. However, the large degree of variation in how — or if — states implemented rigorous math standards makes it hard to determine the true impact of these reforms on math achievement. It takes several years for states and districts to change standards, update textbooks and other instructional materials, and provide professional development on the new learning goals. It takes even longer for teachers to become experts in the standards and know how to use them effectively. What is clear throughout the course of these changes is that states recognized the importance of implementing more rigorous math standards and took steps to raise the bar for their students. But it will take consistent and well-supported implementation over time to make a difference in math achievement.

Ensuring Elementary Teachers are Well-Prepared to Lead Math Instruction

Even the best state math standards are only expectations for learning: standards alone cannot improve instruction. Improvement depends on knowledgeable, engaged teachers and well-designed instructional materials and tools. Ball, Lubienski and Mewborn wrote in 2001 that teachers' mathematical knowledge is key to their ability to solve problems central to their work as educators — problems like choosing tools and instructional materials, making sense of and responding to students' work, and designing useful assignments. And doing these things well requires a deeper level of mathematical knowledge than some may appreciate, even in the elementary and middle grades. It also requires a different type of knowledge than that used by professionals in mathematics-related fields.

> “The teacher has to think from the learner's perspective and to consider what it takes to understand a mathematical idea for someone seeing it for the first time.”
>
> — Ball, Hill & Bass (2005)

Brilliant mathematicians can very well be terrible math teachers — not because they are not able to do math, but because they do not have the knowledge and skills required for teaching it. As *Adding It Up* describes:

> “[Teachers] need to know the mathematics they teach as well as… where it can lead and where their students are headed with it. They need to be able to use their knowledge flexibly in practice to appraise and adapt instructional materials, to represent the content in honest and accessible ways, to plan and conduct instruction, and to assess what students are learning…

A teacher must interpret students' written work, analyze their reasoning, and respond to the different methods they might use in solving a problem. Teaching requires the ability to see the mathematical possibilities in a task, sizing it up and adapting it for a specific group of students… In short, teachers need to muster and deploy a wide range of resources to support the acquisition of mathematical proficiency. ”

Once state math standards were revamped and better aligned, instruction in most states began to concentrate more deeply on fewer topics. But these large-scale changes came with huge challenges for teachers. Teachers had to adapt to more rigorous learning goals and learn to go deeper with instruction to help students master the new standards. In response to "unprecedented requests [from districts] for support" following changes to standards, according to SREB's 2015 Cross-State Findings, state departments of education offered varying levels of support and training to district staff and teachers. Much of this training was made possible by federal grant funding in the early years of state standards implementation and was scaled back in some states after a few years. Where does that leave teachers now?

"Just a darn minute — yesterday you said that X equals **two**!"

Aspiring teachers with strong math backgrounds tend to gravitate toward the secondary grades, where they can exclusively teach math. Results of the 2018 National Survey of Science and Math Education showed that just 3 percent of elementary teachers surveyed held a degree in mathematics or mathematics education, compared with 45 percent of middle grades math teachers and 79 percent of high school math teachers. Because elementary schools consist largely of self-contained classrooms, where one teacher is responsible for providing instruction in all core subjects, elementary teachers must be broadly prepared to teach many subjects. This means their preparation programs must cover a greater variety of subject matter than programs for candidates seeking more specialized secondary credentials. As a result, elementary teacher candidates who enter teacher preparation programs without the type and depth of mathematical knowledge they need to teach math effectively may not gain that knowledge from their programs. This was true when NCES published *Adding It Up* in 2001, and it is still true today.

An **algorithm** is a step-by-step procedure for performing a computation — for example, the process used to add two three-digit numbers.

In 2011, the Center for American Progress argued that teacher candidates in the United States have lower levels of mathematical knowledge, on average, than teachers in countries that perform better on international comparisons of math achievement. Ball, Hill and Bass noted in 2005 that most math teachers in the United States are graduates of the very education system researchers seek to improve — therefore, the authors found it unsurprising that many "lack sound mathematical understanding and skill." For example, in a 2014 review of the scientific literature, Browning and colleagues found that preservice teachers tended to struggle with understanding and representing fractions, understanding the concepts of decimals and place value, and flexibly solving algebraic problems. The authors write that a common theme among the papers examined is that preservice teachers may know how to use algorithms to solve a problem, but they often cannot explain why the algorithms work, a finding reiterated by Reeder and Beteiha in 2016. If a teacher doesn't understand and can't explain why a mathematical process works, it is unlikely that she can fully teach that process to students or troubleshoot a student's difficulties in understanding it or mistakes in applying it.

Teachers Need a Strong Foundation in Mathematics Before Entering the Classroom

For aspiring teachers without deep mathematics knowledge — especially elementary candidates — teacher preparation content is critical. However, *Adding It Up* identified problems with the preparation that preschool to middle school teachers received for teaching math — problems that continue today, according to many researchers.

The National Council on Teacher Quality's 2016 review of undergraduate teacher preparation programs for elementary grades concluded that just 13 percent of the 860 programs examined covered critical math topics, including numbers and operations, algebra, geometry, and data and probability. And in 2018, a similar review of graduate elementary preparation programs found that *just 1 percent* of 201 programs covered these topics. These low rates could help explain why 1 in 4 teacher candidates failed the math portion of a common elementary licensing exam the first time they took it, according to data in NCTQ's 2019 report, *A Fair Chance*.

States oversee educator preparation programs and teacher licensure using several policy levers, including credit-hour or course requirements, licensure exams and teacher preparation standards or competencies. Credit-hour or course requirements in math ensure that candidates complete essential coursework, including courses covering content knowledge and teaching methods. Licensure exams require teacher candidates to demonstrate their content and/or pedagogical knowledge. And teacher preparation standards and competencies explain what states expect their teacher candidates to know and be able to do by the time they complete an educator preparation program. Careful consideration of each policy area can help ensure that beginning teachers are well prepared to enter the classroom and teach mathematics effectively.

Educator Preparation Coursework Requirements for Elementary Mathematics

Linda Gojak, past President of the National Council of Teachers of Mathematics, wrote in 2013 that in addition to general content knowledge, elementary teachers need specialized pedagogical content knowledge in order to use best practices for teaching math. No candidate enters an educator preparation program with that knowledge already in hand; programs must provide courses and experiences that build it.

States can establish requirements beyond core general education requirements for the number and type of math credit hours undergraduate teacher candidates must earn in order to successfully complete their degree. By 2019, seven SREB states had math credit-hour requirements for elementary candidates in traditional preparation programs. Most of these states required that candidates earn either nine or 12 credits in general college-level math courses, such as college algebra, calculus or statistics. However, general courses like these are not focused on developing the specialized content knowledge for teaching or pedagogical knowledge elementary teacher candidates need, as noted by Reeder and Beteiha in 2016.

Of the seven SREB states with general education math course requirements, only two — **Louisiana** and **West Virginia** — also required that elementary teacher candidates earn credit in one or more courses covering mathematics teaching methods or math content knowledge for teachers. And **Louisiana** was the only state to require more than three credit hours in these areas. (See Appendix A.)

In 2008, NCTQ's Mathematics Advisory Group published *No Common Denominator*, a report on the preparation of elementary teachers in mathematics. The eight members of the advisory group agreed with prior recommendations for teacher preparation proposed by other expert groups, including the National Mathematics Advisory Panel, NCTM and the Conference Board of the Mathematical Sciences. The advisory group strongly recommended that elementary teacher candidates take at least three math courses designed specifically for prospective teachers, in addition to one math methods course. These courses should focus on elementary and middle school topics — numbers and operations, algebra, geometry and measurement, and data analysis and probability — and be taught by professors who can connect the math content to elementary classroom instruction.

In 2008, the National Council for Teacher Quality's Mathematics Advisory Group recommended that math courses for elementary teacher candidates in educator preparation programs cover the following topics:

The Breadth of Mathematics Content that Elementary Teachers Need

Critical areas	Essential topics	Estimated class time needed
I. Numbers and operations	1. Whole numbers and place value 2. Fractions and integers 3. Decimals (including ratio, proportion, percent) 4. Estimation	Subtotal: **40 hours**
II. Algebra	5. Constants, variables, expressions 6. Equations 7. Graphs and functions	Subtotal: **30 hours**
III. Geometry and measurement	8. Measurement 9. Basic concepts in plane and solid geometry 10. Polygons and circles 11. Perimeter, area, surface area, volume	Subtotal: **35 hours**
IV. Data analysis and probability	12. Probability and data display and analysis	Subtotal: **10 hours**
Total Estimated Time:		**115 hours** = roughly three 45-hour courses

Source: Adapted from Greenberg and Walsh (2008)

It may be surprising to hear that elementary teachers need content knowledge in areas like algebra, data analysis and probability — topics typically associated with middle school math and beyond. But good math instruction begins to build foundations in these areas very early on. Elementary teachers need to understand not just the content they teach, but also where that content is headed in future grades. Teachers also need to know how to develop students' understandings and thinking patterns to equip them for later learning in math.

Educator preparation programs can and do take heed of recommendations from NCTM, NCTQ, and other groups and independently require that their elementary candidates take courses covering mathematics teaching methods and specialized content knowledge for teaching. But without state requirements this is completely optional, and the number, type and content of these courses can vary widely from program to program (see Appendix B for examples).

In 2007, Kajander asserted that the challenge of ensuring that teachers' understanding of math is conceptual "*must* be faced in [teacher preparation program] methods courses." Kajander's study of preservice teachers found that taking a math teaching methods course that also focused on building teachers' conceptual knowledge of mathematics led to substantial improvements in this knowledge. It also led to changes in prospective teachers' beliefs about how math should be taught. However, the researcher noted that even participants' improved level of conceptual understanding remained "highly inadequate" for teaching. Kajander felt that teacher candidates needed to spend more time building this type of knowledge in order to be well prepared for teaching math. This research makes the case for requiring that the math courses prospective teachers take in undergraduate programs focus on building content knowledge for teaching, not simply ensuring that candidates can perform mathematical operations and solve problems.

> "The challenge of ensuring that teachers' understanding of math is conceptual *must* be faced in teacher preparation program methods courses."

Teacher Licensure Exams for Elementary Education: 16 States, 19 Different Exams

Licensure exams are a policy tool used to ensure that teacher candidates have at least the minimum levels of knowledge and skills required to begin educating students in their chosen fields. In 2019, 13 SREB states required a test of general content knowledge with a stand-alone math score for certification for some or all elementary teacher candidates. (See Appendix A.) A stand-alone score in math, as opposed to a score that combines math items with items from other subjects, helps ensure that teacher candidates demonstrate a basic level of math competency. This kind of exam assesses candidates' ability to *do* the level of math they will be teaching, but it does not assess candidates' ability to *teach* that content.

The math subtest of the Praxis® Elementary Education: Multiple Subjects exam — the most common assessment required for elementary licensure in SREB states — contains 50 items and is designed to assess 67 indicators of general math knowledge and skills, as reported by the test producer. State-developed content exams also tend to assess general knowledge and skills, rather than the deeper level of knowledge necessary for strong teaching. For example, 74 percent of the math subtest of the Elementary Education K-6 Florida Teacher Certification Examination consists of test items targeting general math knowledge, while only 26 percent of the items assess candidates' knowledge of student thinking and instructional practices as they relate to mathematics. Other state-developed content exams in the region are not designed to assess math pedagogy at all.

In general, teacher candidates' performance on licensure exams of content knowledge is only weakly predictive of their performance in the classroom, as summarized by SREB's 2018 commission report, *State Policies to Improve Teacher Preparation*. Studies often do not compare different types of licensure exams, however. A few states have sought alternatives to the usual assessments of general content knowledge for ensuring that elementary teacher candidates have the specialized knowledge and skills needed to teach each subject well, including math. Unfortunately, there are very few licensure exams designed to assess this form of content knowledge.

Maryland requires all elementary teacher candidates to pass the Praxis® Elementary Education: Content Knowledge for Teaching exam rather than a test of general content knowledge. The math subtest of the Content Knowledge for Teaching exam is designed to measure both the general knowledge necessary for doing elementary-level math and the specialized content knowledge teachers need in order to teach the curriculum. According to the test producer, approximately 80 percent of the 52 test items assess this latter type of knowledge. In 2019, North Carolina made this subtest one of two options for elementary candidates. One other SREB state, **Delaware**, allows elementary candidates to choose to take the Content Knowledge for Teaching exam instead of the Multiple Subjects test.

The Wide World of Teacher Licensure Exams

States in the SREB region have selected many different licensure exams to assess elementary teacher candidates' content and pedagogical knowledge. They include:

- Praxis® Early Childhood Education
- Praxis® Education of Young Children
- Praxis® Elementary Education: Multiple Subjects
- Praxis® Elementary Education: Content Knowledge for Teaching
- FTCE Prekindergarten/Primary PK-3
- FTCE Elementary Education K-6
- GACE Early Childhood Education
- Pearson General Curriculum
- OSAT Elementary Education
- Praxis® Elementary Education: Curriculum, Instruction and Assessment
- TExES Core Subjects EC-6
- Praxis® Elementary Education: Content Knowledge
- Praxis® Principles of Learning and Teaching: Early Childhood
- Praxis® Principles of Learning and Teaching: Grades K-6
- FTCE Professional Education Test
- Oklahoma Professional Teaching Examination: PK-8
- TExES Pedagogy and Professional Responsibilities EC-12
- edTPA®
- Praxis® PPAT

How do different exams assess teacher candidates' mathematics content knowledge?

The two questions below are examples of the types of assessment items elementary teacher candidates face on the math subtest of the Praxis® Elementary Education: Multiple Subjects (5001) exam. These items assess a teacher candidate's ability to perform mathematical calculations and draw conclusions, but they are not designed to assess a candidate's ability to teach these concepts to students.

x	y
4	−20
7	−38
12	−68
15	−86

Which of the following equations gives the rule for the table shown ?

(A) $y = -8x + 4$

(B) $y = -7x + 4$

(C) $y = -6x + 4$

(D) $y = -5x + 4$

Jack had three babysitting jobs this week. He worked the same number of hours H on each job. He was paid at a rate of $12 for every hour at his first job, $4 for every half hour at his second job, and $5 for every 20 minutes at his third job. Which of the following expressions could be used to find the total amount, in dollars, Jack earned?

(A) $12 \times H + 4 \times H + 5 \times H$

(B) $12 \times H + 8 \times H + 15 \times H$

(C) $12 \times H + 8 \times H + 20 \times H$

(D) $12 \times H + 4 \times \frac{1}{2} \times H + 5 \times \frac{1}{3} \times H$

In contrast, the next item is an example of the type of assessment item teacher candidates face on the math subtest of the Praxis® Elementary Education: Content Knowledge for Teaching (7801) exam. This item is designed to assess a candidate's ability to interpret a student's mathematical error and identify how that error would generalize to similar problems. Identifying students' mathematical misunderstandings and guiding students to recognize and correct the error in their reasoning is an important part of teaching math.

$$\begin{array}{r} 385 \\ +\,462 \\ \hline 7147 \end{array} \qquad \begin{array}{r} 453 \\ +\,427 \\ \hline 8710 \end{array} \qquad \begin{array}{r} 321 \\ +\,836 \\ \hline 1157 \end{array}$$

Josh is a third-grade student in Ms. Carter's classroom. Josh's answers to three addition problems are shown. He incorrectly answered the first two problems but correctly answered the third problem.

If Josh uses the same strategy to answer the following problem, what will his answer be?

$$\begin{array}{r} 328 \\ +564 \\ \hline \end{array}$$

Source: ETS

Because content exams alone are poor predictors of teacher quality, states have become increasingly interested in assessments of teacher performance and pedagogy. SREB's 2018 Teacher Preparation Commission report notes that practice-based assessments align more strongly with the practical classroom skills teachers need to be effective than do traditional tests for licensure. In 2018, nearly every SREB state required both a test of general content knowledge and an assessment of teacher practice. The Praxis® Principles of Learning and Teaching exam is one assessment that is commonly required by SREB states. This exam is multiple-choice and is designed to assess aspiring teachers' "knowledge and understanding of educational practices," including instruction, professional development and assessment. Florida, Oklahoma, and Texas have developed their own tests of professional teaching skills using a similar format.

Two nationally-available assessments examine teacher practice in greater depth than can a multiple-choice exam: edTPA® and the Praxis® PPAT. Both assessments consist of tasks that require a preservice teacher to submit written commentary, a video of a lesson, and artifacts that provide evidence of the candidate's ability to demonstrate certain types of knowledge and skills. While multiple-choice assessments are typically completed in one testing session that lasts a few hours, edTPA and the PPAT are completed over a period of several months.

"With so many exams available, states must carefully consider what type and depth of knowledge they want elementary teacher candidates to demonstrate for an initial teaching license."

The PPAT is not subject- or grade level-specific and may be completed by a teacher seeking any type of licensure, while edTPA assessments are available for various subjects and grade bands. According to edTPA's developer, the Stanford Center for Assessment, Learning and Equity, about 80 percent of each assessment is designed to measure "pedagogical constructs that underlie the integrated cycle of planning, instruction, and assessment." In addition, the math edTPA assessments examine a teacher candidates' ability to support students' development of conceptual understanding, procedural fluency and problem-solving skills. Candidates must also demonstrate that they can design lessons that include "mathematics-pertinent language demands and supports" and student assessments of math concepts and reasoning skills.

ETS recently created a new teacher practice assessment model designed to allow candidates to demonstrate that they have the knowledge and skills needed for teaching content at the elementary level. Called NOTE™ Elementary Education: Practices for Teaching Content, the assessment is an interactive, computer-based simulation that examines a prospective teacher's ability to model and explain content, lead a group discussion, and elicit student thinking through three language arts and three mathematics tasks. As prospective teachers respond to the assessment's virtual classroom situations, video and audio of their interactions is transmitted to a simulation specialist who controls the simulated students' movements and responses. Assessing teacher candidates' reactions to teaching scenarios in a controlled environment allows for standardized scoring across candidates, though ETS acknowledges that real instructional settings are much more complicated than those provided in the NOTE assessment.

With so many exams available, states must carefully consider what type and depth of knowledge they want elementary teacher candidates to demonstrate for an initial teaching license. Many states will also face the challenge of balancing these desires with the need to grow their teaching workforces. More and more states are requiring pedagogical assessments for licensure. These may be better predictors of teacher practice than a content knowledge exam, but they can also be expensive and time-consuming for both teacher candidates and educator preparation programs.

Standards and Competencies for Elementary Teachers: More Detail, or Less?

The math standards and competencies that prospective elementary teachers are expected to meet by the time they enter the classroom vary even more widely between states than do licensure exam and course requirements. Some states issue guidelines for educator preparation content through state board rule or administrative code, while others allow their departments of education to develop guidelines and use them to determine the content of state licensure assessments.

In 2019, seven SREB states had developed their own math standards or competences for elementary teachers. Six states had no specific standards or competencies these teachers were expected to meet in order to be considered qualified. State-developed expectations range from short and general to very detailed and specific. (See the highlight box on the next page.) States with more detailed guidelines are likely to see more consistency in the content covered by educator preparation programs.

> “States with more detailed standards or competencies for preservice teachers are likely to see more consistency in the content covered by educator preparation programs.”

It is not necessary for states to create their own standards, however. Several expert groups, including AMTE, the Conference Board of the Mathematical Sciences and the Council for the Accreditation of Educator Preparation, have already developed or identified high-quality standards for the knowledge and skills elementary teachers need to teach math effectively. Three states in the SREB region have chosen to adopt some of these existing standards. Arkansas adopted the Association of Mathematics Teacher Educators' 2017 *Standards for Preparing Teachers of Mathematics*, and Georgia based its standards on the Association of Childhood Educators International's 2007 *Elementary Education Standards.* Tennessee used the 2010 NAEYC standards for candidates earning PK-3 licenses and ACEI standards for K-5 licenses. Educator preparation programs in Tennessee must also consult the National Council of Teachers of Mathematics standards, though NCTM currently has no standards for elementary generalist teachers.

The newly revised CAEP 2018 *K-6 Elementary Teacher Preparation Standards* may be among the most thorough of the guidelines developed by expert groups. The CAEP standards are applied to educator preparation programs seeking to earn or maintain CAEP accreditation. Standard 2.b details the knowledge and skills elementary teachers need in the domains of number and operations, operations and algebraic thinking, measurement and data, geometry, and mathematical practices. A different set of standards, AMTE's 2017 *Standards for Preparing Teachers of Mathematics*, lays out the essential content of these domains in even greater detail, albeit in a lengthier and more expository format.

Sources for Detailed Elementary Math Standards and Competencies for Teachers

While some states choose to develop their own standards or competencies for elementary teachers of mathematics, a number of expert groups have published recommendations that states can use. These include:

- AMTE *Standards for Preparing Teachers of Mathematics* (2017)
- CAEP *K-6 Elementary Teacher Preparation Standards* (2018)
- CBMS Essential Grades K-5 Ideas for Teachers, from *The Mathematical Education of Teachers II* (2012)

Comparing Mathematics Standards and Competencies for Elementary Teachers

Some states have brief, general guidelines for the mathematical knowledge and skills they expect elementary teachers to demonstrate by the time they enter the classroom. This leaves open the possibility that educator preparation programs may interpret the guidelines differently and address them to a lesser extent than intended. Other states provide more detailed guidance. Following are examples of how general or detailed these standards can be for elementary teachers' knowledge of math instructional practices.

Examples of less detailed standards in two SREB states:

State 1:

Prior to program completion candidates demonstrate ability to:

(ii) Explain students' strategies while connecting and generalizing ideas, anticipating responses and misconceptions, applying reason, and representing and articulating relationships between mathematical concepts.

State 2:

III. EPPs providing training for elementary and special education general curriculum teachers shall include:

2. Instruction in evidence-based learning trajectories and how to identify what students are able to do and what is needed to address their needs including:

    a. how to identify which mathematical concepts/skills students have and have not demonstrated, and,

    b. how to plan for instruction based on student strengths and needs as determined by the evidence.

Example of more detailed standards in one SREB state:

Knowledge of student thinking and instructional practices

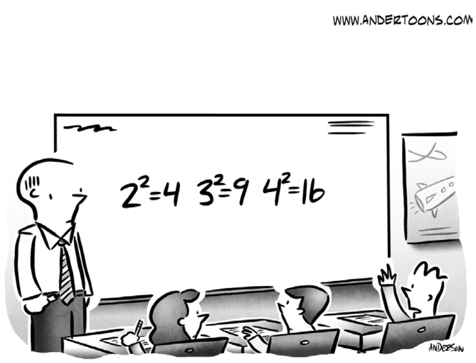
1. Analyze and apply appropriate mathematical concepts, procedures, and professional vocabulary (e.g., subitize, transitivity, iteration, tiling) to evaluate student solutions.
2. Analyze and discriminate among various problem structures with unknowns in all positions in order to develop student understanding of operations (e.g., put-together/take-apart, arrays/area).
3. Analyze and evaluate the validity of a student's mathematical model or argument (e.g., inventive strategies, standard algorithms) used for problem solving.
4. Interpret individual student mathematics assessment data (e.g., diagnostic, formative, progress monitoring) to guide instructional decisions and differentiate instruction.
5. Select and analyze structured experiences for small and large groups of students according to the cognitive complexity of the task.
6. Analyze learning progressions to show how students' mathematical knowledge, skills, and understanding develop over time.
7. Distinguish among the components of math fluency (i.e., accuracy, automaticity, rate, flexibility).

Sources: AL Admin Code Chapter 290-3-3-.06(2)(b)3, NC State Board Policy Manual TCED-011 and FTCE Elementary Education K-6 Competencies and Skills

Whichever math standards or competencies states set for prospective elementary teachers, they should consider whether those guidelines cover the breadth and depth of knowledge elementary teachers need. Guidelines that are less clear and well-defined may result in greater variability between educator preparation programs, and therefore greater variability in the knowledge and skills new teachers bring to the classroom. Ensuring that state standards and competencies for elementary teacher candidates clearly address the key concepts of mathematics identified by researchers and well-qualified groups, including NCTM, CAEP and AMTE, can help states be sure that students in all schools receive high-quality math instruction.

The Importance, Content and Structure of Ongoing Professional Development

Ensuring that elementary teachers are prepared to teach math before they enter the classroom is critical, but even well-prepared teachers should continue to deepen their mathematical understanding as they teach. This was one recommendation from the blue-ribbon Conference Board of the Mathematical Sciences in 2012. And for teachers who did not receive the preparation they deserve from their educator preparation programs, professional development is key to making sure they have the knowledge and skills they need to teach math effectively. Professional development for current elementary teachers should recognize that many do not have strong math backgrounds and focus both on building math content knowledge and on strategies for teaching that content.



"Wait, we're squaring numbers now? We just figured out how to round them!"

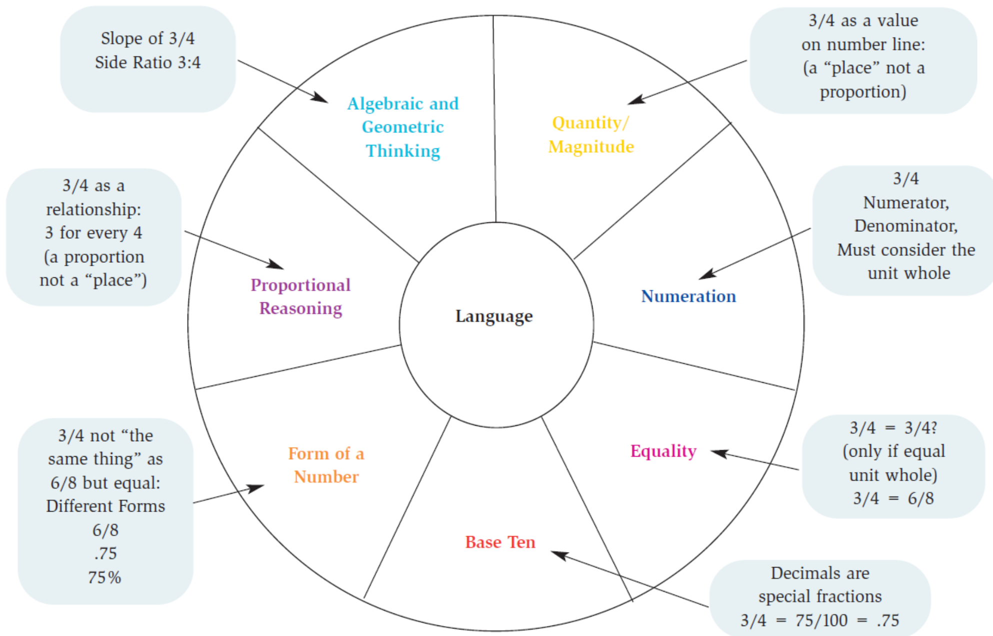
In the elementary grades, much of the mathematical focus is on building a strong foundation for later learning, with topics that include the concept of number (counting, greater than/less than, place value, and so on); the properties of addition, subtraction, multiplication and division; measurement, early geometry, and rational numbers (fractions and decimals). Simply being able to do mathematics at this level may not be difficult for the average adult. But *teaching* these fundamental concepts in a way that leads to full understanding requires mathematical insight and the careful use of mathematical language and terms, wrote Ball, Hill and Bass in 2005.

NCTM's 2014 *Principles to Actions: Ensuring Mathematical Success for All* asserted that too much of math instruction focuses on "learning procedures without any connection to meaning, understanding, or the applications that require these procedures." Elementary teachers who lack strong math knowledge may resort to teaching children common algorithms without understanding why they work or teaching students to memorize shortcuts they themselves learned in school, such as flipping and multiplying when dividing by a fraction. These procedures often get students to a correct answer, but they don't produce the conceptual understanding students need for success in the middle grades, high school and beyond.

> "Professional development for current elementary teachers should recognize that many do not have strong math backgrounds and focus both on building math content knowledge and on strategies for teaching that content."

So what exactly is the knowledge elementary teachers need to gain from professional development? To teach math well, educators need what researchers have termed *mathematical knowledge for teaching,* also known as *content knowledge for teaching.*

The Components of Number Sense and Fractions



Adapted from The Components of Number Sense Model, Faulkner, Hale, Doggett, Cain (2007) found in the Foundations of Math Course (2017). Exceptional Children's Division, North Carolina Department of Public Instruction. Used with permission.

This is not simply a teacher's ability to complete math tasks at the level they teach. Rather, Hill, Rowan & Ball defined this type of knowledge in 2005 as that used to "carry out the work of teaching mathematics." Clements and Sarama characterized math content knowledge for teaching young children as composed of three learning trajectories: "the mathematical content (goal), the developmental progressions of children's thinking and learning, and instructional tasks and teaching strategies that help children move along those progressions." Some researchers have developed frameworks for professional development that help elementary teachers expand their content knowledge for teaching math in the areas most important in the elementary grades, such as the Components of Number Sense and Fractions model featured above. This framework was developed by Cain, Doggett, Faulkner and Hale in 2007 and is currently used by North Carolina's Department of Public Instruction.

Many studies of teachers' mathematical content knowledge for teaching — for example, Hill, Rowan & Ball (2005), Tchoshanov (2011) and Campbell et al. (2015) — find that it is positively related to students' performance in math. It makes sense, then, that building this knowledge before teachers enter the classroom and continuing to grow and refine it through professional development should help teachers improve their instruction — and in turn, their students' math skills and achievement.

Walker wrote in 2007 that professional development should "address the links between **content knowledge, pedagogical knowledge,** and **pedagogical content knowledge**," be **ongoing**, and create opportunities for teachers to **collaborate** and **actively participate** in the process of constructing knowledge. A 2017 Learning Policy Institute review reiterated these important characteristics and identified others that are also associated with effective professional development initiatives, including **modeling effective practice** and providing opportunities for **feedback** and **reflection**. Whatever model is used, professional development is most successful when it is guided by educators with a deep knowledge of math and math instruction.

Districts seeking to provide professional development to their teachers may not always have access to information that helps them identify effective options. This is particularly true of districts that are small, rural or lacking in resources. States can and do assist their districts by providing training directly, but they can also help districts identify effective third-party professional development providers by serving as a clearinghouse for evidence-based options. Louisiana, for example, publishes a yearly guide that identifies vendors who provide training in core curricula. A similar guide could be developed to identify vendors who provide quality training for certain grade levels and subjects. To obtain important feedback on professional development experiences, states could also choose to require that any state dollars spent on teacher training include evaluation of the training.

Design Elements of Effective Professional Development

According to a review conducted by the Learning Policy Institute in 2017, effective professional development for teachers:

- is content-focused;
- incorporates active learning;
- supports collaboration;
- uses models of effective practice;
- provides coaching and expert support;
- offers feedback and reflection; and
- is of sustained duration.

Math Anxiety Can Handicap Teachers and Pass to Students

One factor that can hinder an otherwise well-prepared elementary teacher's ability to teach math effectively is math anxiety, a negative emotional response to mathematics tasks. Foley and colleagues reported in 2017 that math anxiety is a global phenomenon. They and other researchers attribute it to a mix of factors that include poor early math skills, stereotypes based on race or sex, societal pressure, and the transmission of anxiety to children from parents and teachers. Some researchers find that certain teaching methods may be associated with the development of math anxiety. These include a focus on achieving correct answers rather than understanding math concepts, the use of timed tests and speed drills, the use of unrealistic scenarios instead of real-life examples for math problem-solving, and an emphasis on earning high grades in math, according to Adeyemi in 2015.

"Who is putting all the Math books in the Horror section?"

CartoonStock.com

Multiple studies have found that children's math anxiety is associated with that of the adults around them — in other words, math anxiety is contagious. And worse, it has more than just emotional impacts. Neurological research shows that math anxiety actually has negative impacts on individuals' achievement in math. In 2012, Maloney and Beilock cited evidence that the brains of math-anxious children showed hyperactivity in areas responsible for processing negative emotions and reduced activity in areas that support working memory and numerical processing. Math-anxious adults demonstrate worse performance than their peers on tasks that involve counting objects, comparing two quantities and mentally rotating 3D objects. Other research shows that students who view a math test as a challenge to be overcome perform better on the test than those who view it as a threat.

Math anxiety is not limited to K-12 students. In 2011, Hadley and Dorward reviewed past research showing that college students pursuing elementary education majors have some of the highest levels of math anxiety among college students. A study by Ramirez and colleagues in 2018 said that this anxiety extends to teachers' confidence and sense of self-efficacy. Students may pick up on these feelings and internalize them when teachers display fear or frustration with mathematical tasks. These researchers found that teachers' math anxiety was related to their teaching practices, and those teaching practices were related to students' perceptions that their teachers held a fixed mindset belief about math — a belief that one's math ability is inherent and unchangeable. Researchers believe that students may then adopt this mindset themselves, which could explain why teachers' math anxiety has negative impacts on student achievement.

> "A **fixed mindset** belief about math — the belief that one's math ability is inherent and unchangeable — could help explain why math anxiety has negative impacts on student achievement."

Fixed mindsets and stereotypes have a disproportionally large effect on female students. Multiple studies, including one by Beilock, Gunderson, Ramirez and Levine in 2010, have found that having a math-anxious female elementary teacher makes female students more likely to believe the stereotype that boys are better at math than girls, and that girls are better at reading than boys. This is particularly troubling because the vast majority of elementary teachers in the United States are female.

Foley and colleagues wrote in 2017 that addressing math anxiety needs to include a two-pronged approach: find ways to reduce anxiety in those who already experience it, and find ways to prevent it from occurring in the first place. There is currently no consensus on the best way to undertake either effort. Scientists are also still working to determine the causal relationship between math anxiety and poor performance. Some of the most recent data indicate that the two might actually engage in a vicious cycle in which each makes the other worse, said Carey, Hill, Devine and Szucs in 2016.

Still, researchers are working to identify potential interventions that can help students and adults who experience math anxiety. A 2014 study by Park, Ramirez and Beilock found that having students write about the thoughts and feelings they were experiencing before taking a math test helped reduce the performance gap between students with high and low levels of math anxiety. Maloney and colleagues concluded in 2015 that parents with high math anxiety who frequently helped their children with math homework had a detrimental effect on their children's math performance. But telling parents not to help their children with homework is likely to have unintended consequences, too.

A possible alternative was cited in 2018 when Schaeffer and colleagues found that an educational math app could reduce the influence of parents' math anxiety on their attitudes about their children's math potential. The math app was designed to present structured math situations that parents and children could approach together. Researchers found that using the math app did not reduce parents' math anxiety. However, children with higher-anxiety parents who used the math app learned more math over the next three years than did children with low-anxiety parents who did not have access to the app. The authors believe the app provided the family with a positive, structured environment in which to approach math tasks and changed anxious parents' attitudes toward their children's mathematical learning. They believe using the app together helped parents feel that their children were capable of learning math even if parents were anxious about it themselves.

Results like these show that there may very well be ways to address math anxiety for students and parents. Combined with better preparation and training in mathematics for elementary teachers that helps them feel more confident in their teaching, math anxiety interventions may help break the cycle and enable more students to feel they can be successful with math.

Dyscalculia: A Learning Disability in Mathematics

Math performance is malleable, and individuals generally develop better math skills with instruction and practice. However, one group of individuals that faces a distinct disadvantage in learning mathematics is those with **dyscalculia**. Dyscalculia is the less common, less often recognized, and less well-understood cousin of dyslexia, a learning disability in reading. According to leading researchers Butterworth, Varma and Laurillard, dyscalculia is thought to affect about 5 to 7 percent of people. Like dyslexia, dyscalculia seems to be associated with a core deficit in neurological processing: in this case, a deficit in the "foundational capacity for numbers," according to Butterworth in 2010 — not unlike the basic sense of number discussed on page 6 of this report. It is characterized by significant difficulties learning arithmetic.

Individuals with dyscalculia may use their fingers or other tangible objects to count and do simple calculations well beyond the typical age. They may also have difficulty estimating and comparing number quantities. According to Henik, Rubinsten and Ashkenazi, children with dyscalculia struggle to recall arithmetic facts and use basic arithmetic procedures. Butterworth, Varma and Laurillard write that "numbers do not seem to be meaningful for dyscalculics… they do not intuitively grasp the size of a number and its value relative to other numbers. This basic understanding underpins all work with numbers and their relationships to one another." The authors go on to note that "without specialized intervention, most dyscalculic learners are still struggling with basic arithmetic in secondary school."

It is not hard to see how these difficulties might cause enormous challenges in school. And in approximately 20 to 60 percent of children with dyscalculia, it occurs in combination with dyslexia, ADHD or other learning problems, say Henik and colleagues, adding to the barriers to learning these students already face.

How can we help children with dyscalculia?

Scientists are continuing to seek a better understanding of dyscalculia — and a better understanding of how to help those it affects. Butterworth promotes the use of exercises that allow students to practice making the connection between the non-symbolic sense of number and the symbols used to denote numbers — for example, games involving dice and counting. Butterworth, Varma and Laurillard cite activities that involve manipulating concrete representations of number and adaptive software programs that allow students to practice visually representing number quantities. Two such evidence-backed programs that have been in use for a number of years are The Number Race and Graphogame Math. (The Number Race can be downloaded for free from http://www.thenumberrace.com/nr/home.php.)

Bryant noted in 2005 that whole-class math interventions, such as the McGraw-Hill program Number Worlds and Peer Assisted Learning Strategies, have proven effective for low-performing students in math. In a 2007 publication, Michaelson includes a list of general strategies proven to help dyscalculic students, including breaking multi-step problems down into individual steps, the use of diagrams and graphic organizers, and using different colored pens or highlighters to mark operational symbols and parts of questions. Other strategies that research shows can benefit most students who struggle with math include direct, explicit instruction; teachers' verbalization of the cognitive strategies they use to solve problems; and using physical and visual representations of number concepts in conjunction with numerals.

As with dyslexia, students with dyscalculia have the best chance at success if they are identified and provided with intensive intervention as early as possible.

Recommendations

Raising math achievement, like any other state education policy goal, will require sustained efforts and a multifaceted approach. There are no singular solutions to improve early math learning — just as there are no singular solutions in most other policy areas. States taking on the task should consider the following recommendations.

Ensure that elementary teachers are well prepared to teach math by the time they leave educator preparation programs.

1. Establish course requirements for mathematical content knowledge for teaching and mathematics teaching methods. States should require that elementary teacher candidates take courses covering mathematics teaching methods and topics critical to the elementary and middle grades: numbers and operations, algebra, geometry and measurement, and data analysis and probability. These courses should equip teacher candidates with both general knowledge and the specialized knowledge needed to teach these topics to elementary-age students. They should also prepare candidates to build the foundations for math topics that students will study later on.

2. Develop state math standards and competencies for prospective elementary teachers. In states where these already exist, make sure they are detailed and aligned with recommendations from an expert group. These standards and competencies outline the knowledge and skills elementary teachers need to teach math effectively. States with more detailed guidelines are likely to see more consistency in the content covered by educator preparation programs.

3. Examine current state licensure exam requirements and consider whether they adequately assess the knowledge and skills elementary teachers need for the classroom. States that require a test of general content knowledge should consider whether an exam of subject-specific content knowledge for teaching would be a better licensure requirement.

Assist districts in identifying and accessing quality professional development opportunities that deepen elementary teachers' knowledge of math concepts and methods.

1. Ensure that district and school leaders are familiar with evidence-based characteristics of effective professional development. This can help leaders make informed choices when they look at professional development options.

2. Require districts and schools to include evaluation of teacher training opportunities when they use state funds to pay for professional development and report the evaluation results to the state. Evaluation can take many forms, including teacher surveys about the quality of the training they received, classroom observations, and analysis of student assessment data from before and after the training. This feedback is valuable for helping districts and states identify quality professional development opportunities and spend dollars wisely.

3. Provide districts with recommendations for effective professional development opportunities that are aligned with state standards and goals. The information used to make these recommendations could come from state or independent evaluations of training options. It could also incorporate the data obtained from district and school evaluations of the professional development their teachers receive.

SREB Calls for More Research on Dyscalculia and Math Anxiety

Little is currently known about how best to identify dyscalculia and interventions that are effective for students who have it. There is also limited research on how to stop the cycle of math anxiety for individuals who experience it. State leaders may wish to encourage more research — or even partner with research institutions within their states or other organizations — on these issues. SREB will monitor the results of research on dyscalculia to aid in the support of these students.

There are a few steps states can currently take that may help address math anxiety:

1. Carry out a public information campaign to promote a growth mindset toward math and support parents in engaging their young children in simple math-related activities. States can make information and activity ideas available to parents and encourage them to teach their children that math is like playing a sport: practice and effort lead to improvement. Campaign participants may include schools, publicly funded childcare facilities and pediatricians.

2. Require educator preparation programs to promote a growth mindset toward math and other subjects and explicitly address math anxiety in the content knowledge and methods courses taken by teacher candidates. This can help address existing math anxiety in preservice teachers and equip them with the knowledge and skills to prevent their future students from developing it.

3. Ensure that math professional development offered by state departments of education and local education agencies addresses the issue of math anxiety. Professional development is likely the best way to make sure that current teachers know about math anxiety and develop the skills to address it in their classrooms.

Appendix A

Teacher Preparation and Exam Requirements for Elementary Teacher Candidates in Traditional Programs SREB States, 2019

	Credit hours in general college-level mathematics courses	Additional credit hours in mathematics teaching methods or content knowledge for teaching	Teacher preparation standards or competencies for elementary mathematics	Assessment of Performance or General Pedagogy
Alabama	12		**P-3:** Consistent with 2010 NAEYC standards **K-6:** State-developed	edTPA
Arkansas			AMTE	edTPA **or** Praxis Principles of Learning and Teaching: Grades K-6 (5622)
Delaware				edTPA **or** PPAT
Florida			State-developed	FTCE Professional Education Test
Georgia			ACEI	edTPA
Kentucky				Praxis Principles of Learning and Teaching: Grades K-6
Louisiana	**PK-3 License:** 9 **Gr. 1-5 License:** 12	9	State-developed	**PK-3 License:** Praxis Principles of Learning and Teaching: Early Childhood (5621) **Gr. 1-5 License:** Praxis Principles of Learning and Teaching: Grades K-6 (5622)
Maryland	12			**PK-3 License:** Praxis Principles of Learning and Teaching: Early Childhood (5621) **Gr. 1-5 License:** Praxis Principles of Learning and Teaching: Grades K-6 (5622)
Mississippi	9			Praxis Principles of Learning and Teaching: Grades K-6 (5622)
North Carolina			State-developed	edTPA **or** PPAT
Oklahoma	12		State-developed	Oklahoma Professional Teaching Examination (OPTE): PK-8
South Carolina				Praxis Principles of Learning and Teaching: Grades K-6 (5622) **or** edTPA **or** PPAT
Tennessee			**PK-3:** NAEYC **K-5:** ACEI and NCTM	edTPA
Texas	6 to 9		State-developed	TExES Pedagogy and Professional Responsibilities EC-12
Virginia			State-developed	
West Virginia	9	3		Praxis Principles of Learning and Teaching: Grades K-6 (5622) **and** a performance assessment must be developed by each educator preparation program*

* *If an EPP chooses to select a nationally-normed instrument, passing scores on that instrument may substitute for the Principles of Learning and Teaching requirement.*

Source: National Council on Teacher Quality, Pearson, Praxis, edTPA, and SREB analysis of state documents

Appendix A continued

Teacher Preparation and Exam Requirements for Elementary Teacher Candidates in Traditional Programs SREB States, 2019

	Assessment of Subject-Specific Pedagogy or Content Knowledge for Teaching Tests in ***blue*** provide a stand-alone score for mathematics.	Assessment of General Content Knowledge that Includes Math Tests in ***blue*** provide a stand-alone score for mathematics.
Alabama		**P-3 License:** Praxis Early Childhood Education (5025) **K-6 License:** *Praxis Elementary Education: Multiple Subjects (5001)*
Arkansas		**K-6 License:** *Praxis Elementary Education: Multiple Subjects (5001)*
Delaware	**Birth - Gr. 2: License:** Praxis Education of Young Children (5024) **K-6 License Option:** *Praxis Elementary Education: Content Knowledge for Teaching (7801)*	**Birth - Gr. 2: License:** *Praxis Education of Young Children (5024)* **K-6 License Options:** *Praxis Elementary Education: Multiple Subjects (5001)* **or** *Praxis Elementary Education: Content Knowledge for Teaching (7801)*
Florida	†	**Age 3 - Gr. 3 License:** FTCE Prekindergarten/Primary PK-3 **K-6 License:** FTCE Elementary Education K-6
Georgia		GACE Early Childhood Education P-5
Kentucky		*Praxis Elementary Education: Multiple Subjects (5001)*
Louisiana		*Praxis Elementary Education: Multiple Subjects (5001)*
Maryland	**Gr. 1-6 License:** *Praxis Elementary Education: Content Knowledge for Teaching (7801)*	**PK-3 License:** *Praxis Early Childhood Education (5025)* **Gr. 1-6 License:** *Praxis Elementary Education: Content Knowledge for Teaching (7801)*
Mississippi	Praxis Elementary Education: Curriculum, Instruction and Assessment (5017)	Praxis Elementary Education: Curriculum, Instruction and Assessment (5017)
North Carolina	**Option:** *Praxis Elementary Education: Content Knowledge for Teaching Mathematics Subtest (7803)*	*Pearson General Curriculum Mathematics Subtest (203)* **or** *Praxis Elementary Education: Content Knowledge for Teaching Mathematics Subtest (7803)*
Oklahoma		**PK-3 License:** OSAT Early Childhood Education **Gr. 1-8 License:** OSAT Elementary Education Subtests 1 and 2
South Carolina		*Praxis Elementary Education: Multiple Subjects (5001)*
Tennessee	**PK-3 License:** Praxis Education of Young Children (5024) **K-5 License:** Praxis Elementary Education: Curriculum, Instruction and Assessment (5017)	**PK-3 License:** Praxis Elementary Education: Content Knowledge (5018) **K-5 License:** Praxis Elementary Education: Content Knowledge (5018)‡ **or** *Praxis Elementary Education: Multiple Subjects (5001)*
Texas		*TExES Core Subjects EC-6*
Virginia		*Praxis Elementary Education: Multiple Subjects (5001)*
West Virginia		**K-4 License:** Praxis Early Childhood Education (5024) **K-6 License:** *Praxis Elementary Education: Multiple Subjects (5001)*

† *Part of each FTCE content exam for elementary licensure is designed to assess candidates' knowledge of student thinking and instructional practices with respect to a given content area. This includes 23 percent of items on the mathematics subtest of the PK-3 exam and 26 percent of items on the mathematics subtest of the K-6 exam.*

‡ *Accepted through August 31, 2019*

Source: National Council on Teacher Quality, Pearson, Praxis, edTPA, and SREB analysis of state documents

Appendix B

Course Requirements in Highly Ranked vs. Low-Ranked Educator Preparation Programs

NCTQ uses information submitted by educator preparation programs — which includes course syllabi, course requirements and textbooks — to regularly review and rate the programs on their coverage of different content areas. NCTQ's review of undergraduate elementary educator preparation programs includes a letter grade rating for each program's coverage of elementary math. In 2016, NCTQ identified two programs in SREB states that earned an A+ for their coverage of the 12 essential topics identified by the 2008 National Council for Teacher Quality's Mathematics Advisory Group. Below are the math course requirements for these two programs.

Strong Mathematics Course Requirements

College-Level Math	Content Knowledge for Teaching	Teaching Methods
Regional Public University in Georgia		
Math Elective: Recommended MATH 1101 Introduction to Mathematical Modeling or MATH 1111 College Algebra	MATH 2008 Foundations Of Numbers And Operations MATH 3106 Foundations Of Algebra	ECSE 4200 Mathematics Teaching And Curriculum In Grades Pre-K-5
Elective: Recommended MATH 1112 Elementary Statistics or MATH 1200 Plane Trigonometry	MATH 3110 Informal Geometry MATH 3156 Introduction To Data Analysis	
Total Credit Hours: 6	*Total Credit Hours: 12*	*Total Credit Hours: 3*
Private University in North Carolina		
MTH 151 Calculus I	MTH 208 Number and Algebra for K-8 Teachers	EDU 412 Principles of Learning and Teaching II: Mathematics and Sciences Methods and Materials
STS 212 Statistics in Application	MTH 209 Geometry and Data for K-8 Teachers	
Total Credit Hours: 8	*Total Credit Hours: 8*	*Total Credit Hours: 8*

In contrast, the following table contains math course requirements for two programs in the same SREB states that earned ratings of F for their coverage of the 12 essential mathematics topics. This comparison makes it easy to see how widely course requirements and math content coverage can vary, even within the same state.

Inadequate Mathematics Course Requirements

College-Level Math	Content Knowledge for Teaching	Teaching Methods
Private University in Georgia		
MTH 120 Algebraic Modeling or MTH 140 College Algebra or MTH 150 Pre-Calculus	MTH 170 Concepts of Mathematics: Number Concepts	No course specifically addresses teaching methods for mathematics
Additional Math/Science Course		
Total Credit Hours: 3 to 7	*Total Credit Hours: 3*	*Total Credit Hours: 0*
Regional Public University in North Carolina		
One course from options that include College Algebra and Trigonometry, Elementary Statistics, and Pre-Calculus Mathematics	No course specifically addresses content knowledge for teaching mathematics	EDU 3520 Teaching Mathematics
Total Credit Hours: 3	*Total Credit Hours: 0*	*Total Credit Hours: 3*

Sources: NCTQ and publicly available course information for the programs specified

For more information on NCTQ's methodology for rating educator preparation programs, visit https://www.nctq.org/review/how.

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