Math-in-CTE Lesson Plan

Lesson Title:	**Compound Interest – A Millionaire's Best Friend**	*Lesson 01*
Occupational Area:	Business education	
CTE Concept(s):	Compound interest, types of investments, opportunity cost, diversification	
Math Concepts:	Percent, ratio, exponential functions, graphing, rounding, order of operations, estimation, substituting data into formulas and solving, charting, decimals	
Lesson Objective:	The effects of compound interest over time by comparing various investments, calculating compound interest using various rates of return	
Supplies Needed:	Mini whiteboards, dry erase markers, student calculators with exponent button, accompanying worksheet, color pencils/markers, 45 -60 min	

THE "7 ELEMENTS"	**TEACHER NOTES (and answer key)**
1. Introduce the CTE lesson. 1. Hook – As students enter the class, have a variety of drinks and snacks available. Ask them to estimate how much the refreshments cost or ask them to estimate how much they spend on similar items per school day. 2. Vocabulary: a. interest b. principal c. diversification d. liquidity e. exponential functions f. the difference between saving and investing	This will be lesson 2 of 5 in the unit of investing. Previously taught will be unit on the habit of saving and a lesson on the different types of investments. 1. This will tie back in at the end 2. Graph on the whiteboard using Post-It's: "How many of these investing terms are you confident with?" Maybe different color Post-It's for each class • *Chart: bus_bus_charts_01 Element 1* **Things to bring into the discussion somewhere:** **Pay yourself first.** **Traditional company retirement is dissolving, as is Social Security.**
2. Assess students' math awareness as it relates to the CTE lesson.	Use mini whiteboards to gain feedback

1. Jason invests $500 in a savings account at a rate of 1.04% for 1 year. How much is in the account at the end of the year? 2. Evaluate 3x + 5t, when x = 4 and t = 7 3. Convert 3.6% into a decimal.	1. I = Prt I = 500 • 0.0104 = 5.20 The total amount in the account will be $505.20. 2. 3 • 4 + 5 • 7 = 12 + 35 = 47 3. To convert from percent to decimal, we must divide by 100, which simply moves the decimal point two places to the left. (3.6 % = 0.036) Once these two questions are mastered, we will proceed.
3. Work through the math example *embedded* in the CTE lesson. 1. Savings account problem & graph Alicia Martin's savings account principal is $1000. The 2% interest is compounded annually. How much is in the account at the end of the year? At the end of 3 years? 5 years? 10 years? 20 years? Represent your answers via the nearest penny and visually via a line graph.	There are other ways of calculating compound interest – for example, using an Excel spreadsheet or an online investment calculator, – but this method was chosen to give the students a hands-on / "see how it works" approach. Hand out the worksheet to the students. a. Start by solving 1 year and 3 years in the same manner as a repeated simple interest problem. (I = Prt) *Step 1*- figure year 1 interest I = 1000 • 0.02 • 1 = 20 Total value after 1 year is $1020 Add interest to principal for year 2 calculations *Step 2*- figure year 2 interest I = 1020 • 0.02 • 1 = 20.40 Add interest to principal for year 3 calculations *Step 3*- figure year 3 interest I = 1040.40 • 0.02 • = 20.81 *Step 4*- figure total interest after 3 years I = 20 + 20.40 + 20.81= $61.21 Total value after 3 years is $1061.21 *"This process is time consuming and*

	inefficient for anything more than a few compounding periods. There is a better, more efficient way called the compound interest formula." b. Progress into modeling the compound interest formula for 5 years, 10 years, and 20 years: $FV = PV (1 + r/m)^{mt}$, where: • *FV* is the future value, • *PV* is the present value (the principal you start with), • *r* is the annual rate of interest as a decimal, • *m* is the number of times per year the interest is compounded (monthly, annually, etc.), and • *t* is the number of years you leave it invested. *Step 5* – Calculate total value of investment after 5 years. • $FV = 1000 \cdot (1 + 0.02 / 1)^{1 \cdot 5} = 1104.08$ The total value of $1000 invested for 5 years at 2 % compounded annually is $1104.08. *Step 6* – Calculate total value of investment after 10 years. • $FV = 1000 \cdot (1 + 0.02 / 1)^{1 \cdot 10} = 1218.99$ The total value of $1000 invested for 10 years at 2 % compounded annually is $1218.99. *Step 7* – Calculate total value of investment after 20 years. • $FV = 1000 \cdot (1 + 0.02 / 1)^{1 \cdot 20} = 1485.95$ The total value of $1000 invested for 20 years at 2 % compounded annually is $1485.95.

	Step 8 – Graph the results on the provided worksheet – be sure to connect the data points • *Chart: bus_bus_charts_01 Element 3*
4. Work through *related, contextual* math-in-CTE examples. 1. Mutual fund problem & graph Alex Smith's Growth Stock Mutual fund principal is \$1000. The annual rate of 10.5% interest is compounded quarterly. How much is in the account at the end of the year? At the end of 3 years? 5 years? 10 years? 20 years? Represent your answers via the nearest penny and visually via a line graph.	This time we will use the compound interest formula for 1 year, 3 years, 5 years, 10 years, and 20 years: FV = PV (1 + r/m) ^{mt}, where: ▪ *FV* is the future value, ▪ *PV* is the present value (the principal you start with), ▪ *r* is the annual rate of interest as a decimal, ▪ *m* is the number of times per year the interest is compounded (monthly, annually, etc.), and ▪ *t* is the number of years you leave it invested. *Step 1* – Calculate the total value of the investment after 1 year. • Remember we are compounding **quarterly or 4 times per year not just one!** • FV = 1000 • (1 + 0.105 / **4**) $^{\mathbf{4} \cdot 1}$ = 1109.21 The total value of \$1000 invested for 1 year at 10.5 % compounded quarterly is \$1109.21. *Step 2* – Calculate total value of investment after 3 years. • FV = 1000 • (1 + 0.105 / **4**) $^{\mathbf{4} \cdot 3}$ = 1364.70 The total value of \$1000 invested for 3 years at 10.5 % compounded quarterly is \$1364.70. *Step 3* – Calculate total value of investment after 5 years. • FV = 1000 • (1 + 0.105 / **4**) $^{\mathbf{4} \cdot 5}$ = 1679.05

	The total value of $1000 invested for 5 years at 10.5 % compounded quarterly is $1679.05. *Step 4* – Calculate total value of investment after 10 years. • FV $= 1000 \cdot (1 + 0.105 / \mathbf{4})^{\mathbf{4} \cdot 10}$ $= 2819.21$ The total value of $1000 invested for 10 years at 10.5 % compounded quarterly is $2819.21. *Step 5* – Calculate total value of investment after 20 years. • FV $= 1000 \cdot (1 + 0.105 / \mathbf{4})^{\mathbf{4} \cdot 20}$ $= 7947.92$ The total value of $1000 invested for 20 years at 10.5 % compounded quarterly is $7947.92. *Step 6* – Graph the results on the provided worksheet – be sure to connect the data points. •*Chart: bus_bus_charts_01 Element 4*
5. Work through *traditional math* examples. 1. Exponential function problem & graph If $y = a(1 + r)^t$, and y represents the total value after a principal amount (a) is compounded for (t) years at an annual rate of (r) expressed as a decimal (5% will be represented as 0.05), what is the value of y when a = 500, r = 3.5%, and t = 7? Sketch what you think the graph of the equation will look like.	Compound interest is only one example of "exponential growth;" another is bacterial growth: how quickly bacteria grows. "r" and "t" must be in the same units; i.e., if "r" is the annual growth rate then "t" must be the number of years OR if "r" is the monthly growth rate then "t" must be the number of months. 1. $y = a(1 + r)^t$ $= 500\ (1 + 0.035)^7$ $= 500\ (1.035)^7$ $= 636.14$ Make sure to point out all the previous graphs are NOT linear (a straight line) but are in fact exponential as they increase at an increasing rate (get bigger and bigger, faster and faster). By taking the formula and replacing the "+" sign with a "-" sign, we have created an example of exponential decay, which is

	applicable to half-lives of elements, such as how long it takes for uranium or asbestos to become depleted. Another application of exponential functions has to do with fractals; http://www.google.com/search?q=fractals&hl=en&rlz=1T4ADFA_enUS436US436&prmd=ivns&tbm=isch&tbo=u&source=univ&sa=X&ei=BVr6TfL2L4vIsgabveTjAQ&ved=0CGEQsAQ&biw=1362&bih=608 •*Picture: bus_bus_picture_01*
6. Students demonstrate their understanding. Finish the worksheet. 1. CD problem & graph 2. Money Market & graph 3. Savings bonds & graph 4. Single stock & graph 5. Growth stock Mutual fund compounded daily & graph	See the worksheet & key Now, the students should start with the instructions on the worksheet. *Hook* - After the worksheet is completed, bring their attention back to the snack in front of them. What was their estimate of its cost? If it was approximately $3.00 per day times 170 school days (3 • 170 = $510), they do have the ability to invest $500 even if they don't think they do. In fact, if their "snacking habits" are the same for non-school days, some of them could even accumulate $1000 in one year if they discipline themselves for "investing habits."
7. Formal assessment. Which investment will be worth the most at the date of maturity? 1. $14,000 @ 2% for 3 years compounded quarterly 2. $5,000 @ 7% for 15 years compounded annually 3. $10,500 @ 6% for 6 years compounded monthly	 1. $14,863.49 2. $13,795.16 3. $15,036.46

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