# Math-in-CTE Lesson Plan:
# Marketing

| *Lesson Title*: | **Break-Even Point** | Lesson 01 |
|---|---|---|
| *Occupational Area:* | Marketing Ed./Accounting | |
| *CTE Concept(s):* | Fixed Costs, Variable Costs, Total Revenue, Total Costs, Break-Even Point, Pricing Strategies | |
| *Math Concepts:* | Linear Equations, Problem Solving, Tables and Graphing, Interpretation of results | |
| *Lesson Objective:* | At the completion of this lesson students will be able to identify Fixed and Variable Costs as well as compute Total Revenue, Total Costs and Break Even Point. | |
| *Supplies Needed:* | Whiteboard, Pineapple Computers Break-Even Analysis Worksheet, Calculator (graphing calculator optional), Paper and Pencil | |

| THE "7 ELEMENTS" | TEACHER NOTES (and answer key) |
|---|---|
| **1. Introduce the CTE lesson.**<br><br>As part of the analysis process for producing a product, a manager must be able to determine the point where the costs for bringing a product to market are recovered. *This is where the company starts making a **profit**.*<br><br>Costs can be broken down into 2 categories: **fixed costs** and **variable costs**. The point where costs are recovered is called the **Break-Even Point** (BEP). This is the point where **Total Revenue** (TR) is equal to **Total Cost** (TC). **Total Revenue** is the product selling price (P) times the quantity (Q) of products sold. **Total Cost** (TC) is fixed costs (F) plus variable costs (V). | As you identify various terms you may write the terms on the board along with their applicable equations.<br><br>**Profit (P)**: Revenue - Costs<br><br>**Fixed costs (F)**: These are costs that do not vary with the number of units produced. These costs might include rent, salaries, building costs, machinery costs, etc.<br><br>**Variable costs (V)**: These are costs that will vary with the number of units produced. These costs will include raw material costs, packaging costs and any other cost that will change or vary with the number of units produced. (V/unit • #units)<br><br>**Break Even Point**: Total Revenue = Total Costs |

| | BEP=P • Q = F+(V • Q) |
|---|---|
| | **Total Revenue**: P • Q |
| | **Total Costs**: F + (V • Q) |

**2. Assess students' math awareness as it relates to the CTE lesson.**

One of the ways that you can estimate the break-even point is to construct a table for total revenue and total costs and see where the values for total revenue and total cost are the same. The values for quantity for TR and TC will give you a range for the break-even point.

**For example**: Given a selling price of $5, calculate the total revenue for each quantity in the table below.

TR = P • Q

TR= $5 • Q

| Q | 5 | 10 | 15 | 20 |
|---|---|---|---|---|
| TR ($) | | | | |

Given a fixed cost of $30 and a variable cost of $3, calculate the total cost for each quantity in the table below.

TC = F + (V • Q)

TC = $30 + ($3 • Q)

| Q | 5 | 10 | 15 | 20 |
|---|---|---|---|---|
| TC ($) | | | | |

Purpose of this step is to assess the student's ability to construct a table and create a graph from that table.

In this step, the teacher would construct the following formulas, tables and graphs on the board and as a class fill in the tables based on the results of their calculations for each applicable formula.

Total Revenue

| Q | 5 | 10 | **15** | 20 |
|---|---|---|---|---|
| TR ($) | 25 | 50 | **75** | 100 |

Total Cost

| Q | 5 | 10 | **15** | 20 |
|---|---|---|---|---|
| TC ($) | 45 | 60 | **75** | 90 |

Plot the total revenue and total costs on the same graph. Remember that the X-axis is generally the independent variable. What would be the independent variable in our example? (**Ans**: Q or Quantity). If the X-axis is the independent variable, what then is the Y-axis? (**Ans**: The dependent variable or $ in this example).

Plotting the TR and TC will give you a graphical view of where the break-even point occurs. The area between the TR and TC lines to the left of the point of intersection represents the costs to bring the product to market. The area between the TR and TC lines to the right of the point of intersection represents profit.

(x-axis = Quantity, y-axis= $)


To get the exact break-even point you will need to solve the TR = TC equation. Solve the following equation for Q using the values, P=$5, F=$30 and V=$3:
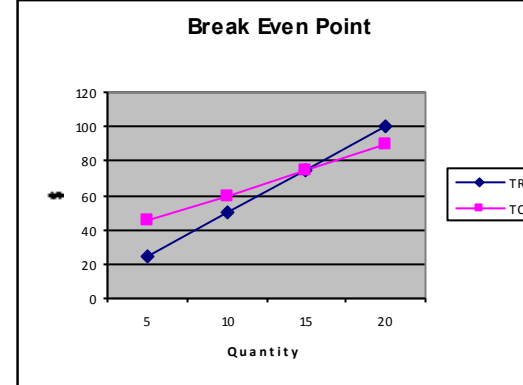
$$P \cdot Q = F + (V \cdot Q)$$

Be sure to check your answer.


What have you noticed?

When does total revenue equal total cost?

What quantity of products do we have to sell in order for our

**Break Even Point**

(Graph: x-axis = Quantity, y-axis = $, lines labeled TR and TC)

The final step of the demonstration is to solve the equation for BEP. The following process can be done by the teacher on the board.

Substituting in the values for P, F and V
$$5Q = 30 + 3Q$$
$$5Q - 3Q = 30 + 3Q - 3Q$$
$$2Q = 30$$
$$2Q / 2 = 30 / 2$$
$$\textbf{Q = 15}$$
Check:
$$5(15) = 30 + 3(15)$$
$$\textbf{75 = 75}$$

Student answers will vary.

When total revenue and total costs are $75.

When quantity is 15.

| | |
|---|---|
| total revenue and total costs to be equal? | |
| **3. Work through the math example *embedded* in the CTE lesson.**<br><br>So far we have talked a little bit about fixed costs, variable costs, and break-even points. I have shown you three ways (constructing a table, graphing values and solving an equation) to determine the quantity of product that must be sold in order to recover our costs, or the break-even point.<br><br>Let's go through a typical scenario that you will likely come across in your visit to the company board room.<br><br>**Scenario**:<br><br>You are the marketing manager for Pineapple Computers and are in charge of launching a new product – the Y-Phone. So far the company has invested $250,000 in a new plant to produce the Y-Phone. Each Y-Phone costs the company $10 to build, and your customer focus groups have told you that you can sell the Y-Phone for no more than $200 each.<br><br>Mr. James Buffet, CEO of Pineapple Computers, asks you to tell the Board of Directors how many Y-Phones must be sold to recover production costs and profitability will occur. The Board would like to see a graph depicting the break-even point and they would like to have the exact quantity value.<br><br>Your task is to construct a graph showing total revenue and total cost with a label identifying the exact value of the break-even point.<br><br>Pass out Pineapple Computers – Break Even Analysis | The purpose of this example is to demonstrate the importance of problem-solving using mathematical concepts in relevant, marketing scenarios.<br><br>**Supplies**: *Pineapple Computers Break-Even Analysis* Worksheet, Calculator<br><br>The teacher will pass out the Pineapple *Computer Break- Even Analysis* worksheet to each student. This can be an individual or small group activity. After a satisfactory period of time, the teacher will work through the worksheet as a class – be sure to involve as many students as possible in the completion of the worksheet. Remember to circulate about the classroom, checking for understanding, during the activity time. |

| worksheet. | |
|---|---|
| **4. Work through *related, contextual* math-in-CTE examples.** Let's do a couple more break-even point calculations. | The purpose of this step is to provide the students with additional practice in calculating break-even point using the BEP formula: $BEP = \dfrac{FC}{P - VC}$. **Supplies**: Calculator The teacher will work these problems with the students on the board. |
| 1. Bob has a lawn mowing business and invests $300 in a new lawn mower for his business. He uses 1 tank of gas, costing $10, on each yard he mows. Bob charges his customers $35 to mow their yards. How many yards must Bob mow in order to begin making a profit? | 1. F = $300, V = $10, P = $35 $$BEP = \frac{FC}{P - VC}$$ $$BEP = \frac{300}{35 - 10}$$ $$BEP = 12$$ |
| 2. Karen makes custom jewelry. In order to sell her jewelry, Karen rents a small kiosk at the local mall for $50 per month. She must buy blank medallions, stones, and chains to make her custom necklaces. These supplies cost approximately $30. Karen sells her necklaces for $55 each. How many necklaces must she sell to cover her costs? | 2. F = $50, V = $30, P = $55 $$BEP = \frac{FC}{P - VC}$$ $$BEP = \frac{50}{55 - 30}$$ $$BEP = 2$$ |

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| **5. Work through *traditional math* examples.**<br><br>Now let's practice solving equations on our own.<br><br>(Pass out *Math Equation* worksheet) | The purpose of this step is to reinforce the student's knowledge of solving mathematical equations.<br><br>At the teacher's discretion, the students may work individually or in small groups. After an adequate period of time, collect the worksheets for grading. Remember to circulate about the room, check for understanding, and re-teach as necessary.<br><br>**Supplies**: *Math Equation* Worksheet, Calculators<br><br>The *Math Equation* Worksheet may be obtained from:<br><br>[WWW.CPM.ORG](WWW.CPM.ORG) Algebra 1 Connections-Skill Builder #11 |
| **6. Students demonstrate their understanding.**<br><br>Now let's have a little fun with our break-even point formula.<br><br>I would like you to create your own problem and exchange it with your neighbor to solve. You will make up a company and tell us about the product you are selling (be creative). In addition, you will tell us the selling price for the product, the fixed costs, and variable costs for your product, and your neighbor will state the break-even point. Express the break-even point as a complete sentence. For example: *The break-even point for the Y-Phone is 1316 units.* Write your name at the top of your paper and the problem-solver's name by the break-even point problem solution. When you are done, turn in your work.<br><br>So, today we have talked about profit, fixed costs, variable costs, selling price, and break-even point. We have used our math skills to calculate how many units of product we must | The purpose of this step is to allow the students to express their creativity as well as, demonstrate their ability to calculate the break-even point.<br><br>During the activity, the teacher should circulate about the room checking for understanding, re-teaching and coaching as necessary. After an adequate period of time, collect the work.<br><br>**Supplies**: Paper, Pencil, Calculator |

sell at a given price, with given fixed and variable costs, in order to recover our startup costs and begin making a profit. As a manager, you will make decisions about the viability of a product using break-even point analysis, as well as a variety of other metrics.

| **7. Formal assessment.** | The purpose of this problem is to assess the students understanding of the fixed costs, variable costs, total revenue, and total cost, as well as break-even point. |
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**7. Formal assessment.**

You are the product line manager for Sound on the Move, a car stereo component manufacture. Sound on the Move is venturing into the motorcycle market with a new product called Vroooom Tunes. Your development costs are fixed at $50,000 with raw material and labor costs at approximately $125 per unit. From your competitive analysis and focus group feedback, you have determined that the selling price for Vroooom Tunes should be $325. As part of the justification process, you will need to tell the President of Sound on the Move how many units will need to be sold before the product is profitable.

Use the BEP formula to calculate the number of units that must be sold in order to recover the costs and start making a profit. Express your answer in a complete sentence.

(Optional: Graph Total Revenue and Total Cost and label the BEP)

The purpose of this problem is to assess the students understanding of the fixed costs, variable costs, total revenue, and total cost, as well as break-even point.

**Supplies**: Calculator

$F = \$50,000$

$V = \$125$

$P = \$325$

$$BEP = \frac{FC}{P-VC}$$

$$BEP = \frac{50,000}{325-125}$$

$$BEP = 250$$

Sound on the Move will need to sell 250 units of Vroooom Tunes in order to recover their costs and begin to make a profit.
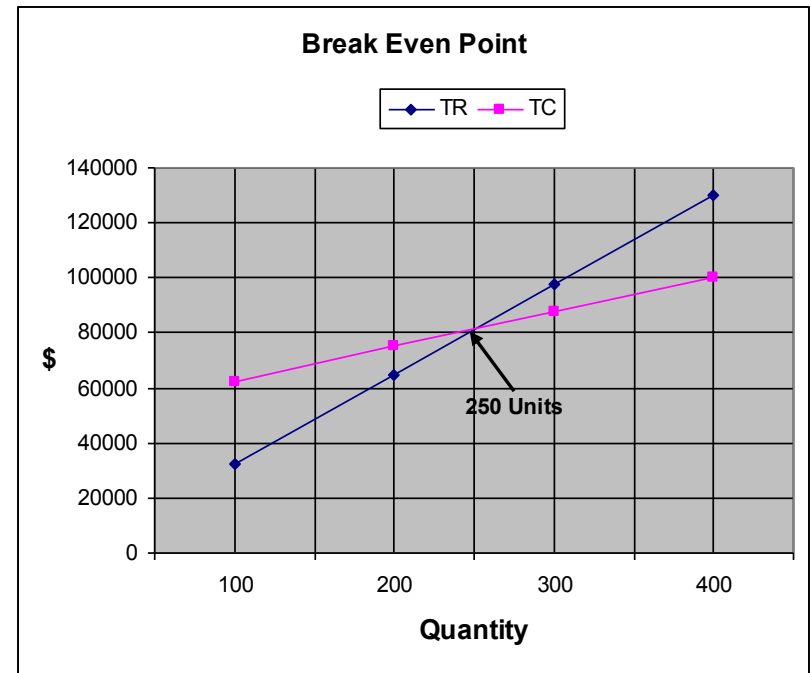
Optional:

$TR = P \cdot Q$

| Q | 100 | 200 | 300 | 400 |
|---|---|---|---|---|
| TR | 32500 | 65000 | 97500 | 130000 |

TC = F + (V • Q)

| Q | 100 | 200 | 300 | 400 |
|------|-------|-------|-------|--------|
| TC | 62500 | 75000 | 87500 | 100000 |

**Break Even Point**



NOTES: